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# METHOD FOR CALCULATING CONVECTIVE HEAT-TRANSFER COEFFICIENTS OVER TURBINE VANE SURFACES

by Daniel J. Gauntner and James Sucec\*

Lewis Research Center

#### SUMMARY

A method for calculating laminar, transitional, and turbulent convective heat-transfer coefficients over the surfaces of turbine vanes is described. Results of calculations with an approximate integral solution method are compared with results from a turbulent flat plate solution and a finite-difference solution.

The approximate integral solution method produced heat-transfer coefficients in good agreement with the finite-difference solution method in the laminar and fully turbulent flow regimes. The transitional flow regime length was approximately the same although the regime began earlier for the finite-difference solution. The turbulent flat plate solution produced a conservative heat-transfer distribution which exceeded the finite-difference solution by as much as 150 percent.

The calculations were made for given gas stream conditions of turbine inlet temperature equal to 1273 K, turbine inlet pressure of 45.7 newtons per square centimeter, a fuel/air ratio of 0.0, a vane channel inlet Mach number of 0.31, and an exit Mach number of 0.84. Transition criteria were based on momentum thickness Reynolds numbers in the integral solution.

#### INTRODUCTION

In the design of blades and vanes for gas turbine engines, the convective heattransfer rate from the gas stream to the surface must be calculated. The simplest approach is to use the flat plate solution. The flat plate solution, however, applies only to zero pressure gradient flow. In many instances, though, this approximation leads to

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satisfactory results for initial calculations because the Stanton number is relatively insensitive to pressure gradient (ref 1). One of the most powerful methods of calculating the heat flux to a turbine blade is to solve the boundary layer mass, momentum, energy, and turbulent energy equations simultaneously by a finite-difference approach (ref. 2). In between these two approaches is the approximate integral solution. This method is more general than the flat plate solution, but not as powerful as the finite-difference methods.

An approximate integral solution (ref. 3) for the local surface heat-transfer coefficient that takes into account the effect of pressure gradient and of variable surface temperature is reviewed in this report. The turbulent and transitional heat-transfer coefficients are based on the approximate solutions of the equations for the thermal boundary layer. The laminar coefficients are those due to reference 4 with the corrections for variable wall temperature due to reference 5. The boundary layer transition criteria used in determining the heat-transfer coefficients are based on the boundary layer momentum thickness Reynolds number. Reference 6 discusses data from the literature and presents an empirical expression relating the start of transition to the critical momentum thickness Reynolds number. These Reynolds numbers are presented as a function of a gas stream pressure gradient parameter and the gas stream turbulence intensity level.

#### SYMBOLS

A <sub>o</sub>	constant
В	constant, A <sub>O</sub> Pr <sup>-n</sup>
b	variable used in eq. (5)
C <sub>p</sub>	specific heat
D	diameter of leading edge of vane
Eu	Euler number, - $(dP_g/dx)/(\rho_g u_g^2/x)$
F <sub>lam</sub>	$Nu/Re_{\mathbf{x}}^{0.5}$
$h_g$	heat-transfer coefficient
k	thermal conductivity
m,n	exponents
Nu	Nusselt number
P	pressure

Pr Prandtl number

R radius of leading edge

Re Reynolds number

 $St_{x}$  local Stanton number,  $h_{g_{x}}/\rho_{s}C_{p}u_{s}$ 

T temperature

T<sub>\*</sub> temperature arbitrarily taken from vane temperature distribution

Tu turbulence level

Tu mean turbulence level

u<sub>s</sub> local free-stream velocity

 $V_{cr}$  local critical velocity

x local distance from leading edge of vane

 $\delta_{\tau}^*$  enthalpy thickness

 $\theta$  momentum thickness

λ pressure gradient parameter

μ dynamic viscosity

ν kinematic viscosity

 $\rho$  density

Subscripts:

crit critical location of flow transition

ge effective gas

g<sub>x</sub> local gas

ges condition for eq. (3)

lam laminar flow conditions

m metal

rat correction factor on constant surface temperature coefficient

ref reference

s static

T<sub>i</sub> turbine inlet

t turbulent

w wall

x local condition

0 refers to positions where flow ceases to be laminar or where it becomes fully turbulent

#### ANALYSIS

The design of turbine vanes and blades for gas turbine engines requires accurate convective heat-transfer coefficients around the entire airfoil surface. Around the exterior cylindrical leading edge of the vane, the cylinder correlation is commonly used. For the remaining surfaces of the vane, laminar, transition, and turbulent coefficients must be supplied.

#### Leading Edge Region

Around the leading edge of a turbine vane the heat-transfer correlation for a cylinder in cross flow (ref. 7) gives

$$h_g = 1.14 \text{ Re}^{0.5} \text{Pr}^{0.4} \frac{k}{D} \left[ 1 - \left( \frac{|\theta|}{90} \right)^3 \right] \quad 0 < |\theta| < 80^0$$
 (1)

where Re =  $\rho u_s D/\mu$  and  $\rho$  is determined from the perfect gas law. All properties for the cylinder correlation are evaluated using air properties at the reference temperature (ref. 8):

$$T_{ref} = 0.5 T_m + 0.23 T_s + 0.22 T_{ge}$$
 (2)

Recent tests have shown that leading edge turbulence could increase the heat-transfer coefficient by a factor of up to 1.8 (ref. 9). This effect can be estimated (ref. 10) if the turbulence characteristics are known. The correction for this effect, called an augmentation factor, acts as a factor multiplying equation (1).

#### Laminar Flow Region

For the laminar flow region, the local convective heat-transfer coefficients are obtained from previous results (ref. 4):

$$h_{g_{X}} = \overline{F}_{fam} Re_{X}^{0.5} Pr^{1/3} \left[ \frac{k}{x} \right]$$
 (3)

where

$$\overline{F}_{\text{fam}} = \frac{Nu}{Pr^{1/3}Re_x^{0.5}}$$

$$Re_{x} = \frac{\rho u_{s}x}{\mu}$$

The  $\overline{F}_{fam}$  Pr $^{1/3}$  =  $F_{fam}$  is evaluated as a function of the local Euler number. Eu and the local ratio of  $T_s/T_m$ . The values of  $F_{fam}$ , or  $Nu/Re_x^{0.5}$ , are approximated by solutions for wedge-type flow. These wedge solutions may be used to approximate heat-transfer coefficients along an arbitrary profile. The value of Eu at any position along the profile determines the corresponding wedge for which, at the same distance x from the stagnation point, the heat transfer on the wedge and the arbitrary profile are assumed to be equal.

The convective heat-transfer coefficients obtained from equation (3) are based on constant wall temperature. Since turbine blades have varying wall temperatures, it is necessary to correct the convective coefficients to account for such variations. A method for making such a correction exists (ref. 5). The method determines the ratio of the variable surface temperature coefficient to the constant surface temperature coefficient:

$$h_g = h_{rat}h_g (eq. (3))$$

where h rat is a function of b and b is determined from

$$\frac{T_{ge} - T_{m}}{T_{ge} - T_{*}} \propto x^{b}$$

and  $T_m$  is the local surface temperature,  $T_*$  is a reference wall temperature, and x is the distance from the stagnation point. One set of data of  $h_{rat}$  against b has been curve fit as follows (ref. 5):

$$h_{rat} = -0.111 b^2 + 0.318 b + 1.0 \quad C \le b \le 0.6$$
 (5a)

$$h_{rat} = -0.520 \text{ b}^2 + 0.310 \text{ b} + 1.0 \quad -0.5 \le \text{b} < 0$$
 (5b)

$$h_{rat} = -4.977 b^2 - 4.288 b - 0.186 -0.76 \le b < -0.5$$
 (5c)

#### Turbulent Flow Region

The technique for predicting turbulent local convective heat-transfer coefficients is based on the approximate integral solution (ref. 3). The solution of the equation for the thermal boundary layer takes into approximate account the effects of thermal history, free stream velocity variation, and free stream density variation.

In the solution, the integral energy equation was arranged into a form involving the local enthalpy thickness  $\delta_{\tau}^*$ :

$$\frac{d\delta_{\tau}^*}{dx} + \left[ \frac{1}{\rho_s} \frac{d\rho_s}{dx} + \frac{1}{u_s} \frac{du_s}{dx} + \frac{1}{(T_w - T_s)} \frac{d(T_w - T_s)}{dx} \right] \delta_{\tau}^* = \frac{h_x}{\rho_s C_p u_s}$$
(6)

If a relation could be found between  $\operatorname{St}_{\mathbf{X}} = \operatorname{h}_{\mathbf{X}}/\rho_{\mathbf{S}} \operatorname{C}_{\mathbf{p}} \mathbf{u}_{\mathbf{S}}$  and  $\delta_{\tau}^*$ , equation (6) could be solved for  $\delta_{\tau}^*$  as a function of  $\mathbf{x}$  and therefore  $\operatorname{h}_{\mathbf{X}}$  as a function of  $\mathbf{x}$ . For a zero pressure gradient, constant property flow over an isothermal flat plate, the Stanton number has the following form:

$$St_{x} = A_{o}Re_{x}^{-m}Pr^{-n}$$
 (7)

The values of A<sub>0</sub>, m, and n depend on whether the flow is laminar, transitional, or turbulent. Substituting equation (7) into equation (6) gives the functional form of the Stanton number for the flat plate.

$$St_{x} = B^{1/(1-m)}(1-m)^{-m/(1-m)}Re_{\delta_{x}^{*}}^{-m/(1-m)}$$
 (8)

where  $B = A_0 Pr^{-n}$ . Furthermore, the  $St_x$  in equation (6) was proposed to be some general function of  $Re_{\delta^*}$  but otherwise independent of body shape, surface temperature variation, free-stream velocity, and density variation. If this is true, a universal relationship between  $St_x$  and  $Re_{\delta^*}$  must be given by equation (8). If equation (8) is substituted into equation (6) and the resulting expression integrated, the following result will be applicable:

$$\frac{h_{x}}{\rho_{s}C_{p}u_{s}} = \frac{B^{1/(1-m)}(1-m)^{-m/(1-m)}\left[\mu_{s}(T_{s}-T_{w})\right]^{m/(1-m)}}{\left\{B^{1/(1-m)}(1-m)^{1/(m-1)}\int_{x_{0}}^{x} \frac{\rho_{s}u_{s}(T_{s}-T_{w})^{1/(1-m)}}{\mu_{s}^{m/(m-1)}} dx + \left[\rho_{s}u_{s}(T_{s}-T_{w})\delta_{\tau}^{\bullet}\right]_{x_{0}}^{1/(1-m)}\right\}^{m}}$$
(9)

where  $x_0 = x_{crit}$ 

To evaluate B and m in equation (9), the expression for the turbulent local convective heat-transfer coefficient for a flat plate is used; namely,

$$h_{g_{x}} = 0.0296 \text{ Re}_{x}^{0.8} \text{Pr}^{1/3} \frac{k}{x}$$
 (10)

Expressing equation (10) in the form of equation (8) gives  $B = 0.0296 \text{ Pr}^{-2/3}$  and m = 0.2. Assuming the free-stream dynamic viscosity  $\mu_{\rm B}$  is only a weak function of x,

$$\frac{h_{g_{X}}}{\rho_{s}^{C} p^{u_{s}}} = \frac{0.0296 \text{ Pr}^{-2/3} \left(T_{s} - T_{w}\right)^{0.25} \left(\frac{T_{s}}{T_{w}}\right)^{0.25}}{\left[\int_{x_{0}}^{x} \frac{\rho_{s}^{u_{s}} (T_{s} - T_{w})^{1.25}}{\mu_{s}} dx + \left[\frac{0.8}{0.0296 \text{ Pr}^{-2/3}} \left(\frac{\rho_{s}^{u_{s}} \delta_{7}^{\bullet}}{\mu_{s}}\right) (T_{s} - T_{w})\right]_{x_{0}}^{1.25}\right]^{0.2}}$$
(11)

The factor  $(T_s/T_w)^{0.25}$  in the numerator of the right side of equation (11) is the temperature ratio correction for property variation across the boundary layer (ref. 11) for  $T_w/T_s < 1$ . The temperature-dependent fluid properties, then, are evaluated at the local free-stream static temperature.

#### Transition Flow Region

Over some portions of the surface of a turbine vane, the gas stream flow fluctuates intermittently between laminar and turbulent flow. This region of transition is not really described accurately by either equation (4) or equation (11). Thus, an empirical expression analogous to the form of equation (10) is sought which is applicable in the tran-

sition region. Experimental results for heat transfer in the transition region along a flat plate have been obtained (ref. 3). For Reynolds numbers  $\mathrm{Re}_{\mathbf{X}} > 200~000$  the heat-transfer correlation in the transition region had the form

$$Nu_{x} = 0.000386 \text{ Re}_{x}^{10/9}$$
 (12)

Assuming again the form of equation (8), with Pr=0.7, the values  $B=0.000435\ Pr^{-2/3}$  and m=-1/9 result. When these values of B and m are substituted into equation (9), an expression of  $h_{g_X}$  as a function of x in the transition region results:

$$\frac{h_{g_{X}}}{\rho_{s}C_{p}u_{s}} = \frac{0.000435 \text{ Pr}^{-2/3} \left(T_{s} - T_{w}\right)^{-0.1} \left(\frac{T_{s}}{T_{w}}\right)^{0.25}}{\left[\int_{x_{0}}^{x} \frac{\rho_{s}u_{s}(T_{s} - T_{w})^{0.9}}{\mu_{s}} dx + \left[\frac{1.111}{0.000435 \text{ Pr}^{-2/3}} \left(\frac{\rho_{s}u_{s}\delta_{\tau}^{*}}{\mu_{s}}\right) \left(T_{s} - T_{w}\right)\right]_{x_{0}}^{0.9} -0.111} (13)$$

To use equations (11) and (13) it is necessary to evaluate  $\delta_{\tau}^*$  at  $x_0$ . Since  $\delta_{\tau}^*$  is continuous at  $x_0$  (between laminar and transitional and between transitional and turbulent) it is possible to evaluate  $\delta_{\tau}^*$  by rearranging equation (8):

$$\left(\frac{\rho_{\mathbf{S}} \mathbf{u}_{\mathbf{S}} \delta_{\tau}^{*}}{\mu_{\mathbf{S}}}\right)_{\mathbf{X}_{0}} = \frac{\mathbf{B}^{1/\mathbf{m}}}{1 - \mathbf{m}} \left(\frac{\mathbf{h}_{\mathbf{g}_{\mathbf{X}}}}{\rho_{\mathbf{S}} \mathbf{C}_{\mathbf{p}} \mathbf{u}_{\mathbf{S}}}\right)_{\mathbf{X}_{0}}^{1 - \frac{1}{\mathbf{m}}} \tag{14}$$

At  $x_0$ , the values of m, B, and  $h_{g_x}$  are from the laminar and transitional expressions. They are used in equations (13) and (11), respectively. For equation (11), B = 0.000435 Pr<sup>-2/3</sup> and m = -1/9. For equation (13), B = 0.332 Pr<sup>-2/3</sup> and m = 0.5 when  $\overline{F}_{fam}$  = 0.332 in equation (3).

Equations (11) and (13) are finally rewritten in useful form giving for the turbulent  $^{\rm h}{\rm g}_{\rm X}$ 

$$h_{g_{X}} = \frac{\rho_{s}C_{p}u_{s} (0.0296) Pr^{-2/3} (T_{s} - T_{w})^{0.25} \left(\frac{T_{s}}{T_{w}}\right)^{0.25}}{\frac{\rho_{s}u_{s} (T_{s} - T_{w})^{1.25}}{\mu_{s}} dx + \left[\frac{0.8 (0.9) (0.000435)}{0.0296} \left(\frac{\frac{h_{g_{X}}}{\rho_{s}C_{p}u_{s}}}{0.000435 Pr^{-2/3}}\right)^{10} (T_{s} - T_{w})\right]_{x_{0}}^{1.25}}$$

and for the transitional hg

$$\frac{\rho_{s}C_{p}u_{s}}{\left\{ \int_{x_{0}}^{x} \frac{\rho_{s}u_{s}(T_{s} - T_{w})^{0.9} (0.000435) Pr^{-2/3} (T_{s} - T_{w})^{-0.1} \left(\frac{T_{s}}{T_{w}}\right)^{0.25}}{\frac{\rho_{s}u_{s}(T_{s} - T_{w})^{0.9}}{\mu_{s}} dx + \left[ \frac{1.111 (2) (0.332)^{2} Pr^{-2/3}}{0.000435 \left(\frac{h_{g_{x}}}{\rho_{s}C_{p}u_{s}}\right)} (T_{s} - T_{w}) \right]_{x_{0}}^{0.9} \right\}^{-0.111}}$$

#### Transition Criteria

The change from laminar to transitional flow and from transitional to turbulent flow occurs when the Reynolds number becomes sufficiently high and thus allows instabilities in the boundary layer to grow. To account for local variations in the boundary layer, a Reynolds number based on the momentum thickness is used to determine the values of  $x_0$  in equations (15) and (16) where changes between flow regimes occur. An approximate expression for the momentum thickness can be used (rcf. 12):

$$\theta = \frac{0.67 \ \mu_{s}^{0.5}}{\rho_{s}^{0.5} u_{s}^{3}} \left( \int_{0}^{x} u_{s}^{5} dx \right)^{0.5}$$
 (17)

Thus, the Reynolds number based on momentum thickness can be found as a function of x:

$$Re_{\theta} = \frac{\rho_{s} u_{s}}{\mu_{s}} \left[ \frac{0.67 \ \mu_{s}^{0.5}}{\rho_{s}^{0.5} u_{s}^{3}} \left( \int_{0}^{x} u_{s}^{5} dx \right)^{0.5} \right]$$
(18)

The values of  $\mathbf{x}_0$  (where changes occur from laminar to transitional flow) can then be found from equation (18). This equation is assumed to apply also in the transitional regime and up to the location where the flow goes from the transitional to turbulent. The question arises as to what to use for  $\mathrm{Re}_{\theta}$   $\mathbf{x}_0$ . Empirical expressions are given (ref. 6) for the value of  $\mathrm{Re}_{\theta}$  at the start of transition (that is, the change from laminar to transitional flow) for a zero pressure gradient case and a nonzero pressure gradient case. They are, respectively,

$$Re_{\hat{g}} = 190 + exp (6.88 - 1.03 Tu)$$
 (19)

$$Re_{\theta} = [0.27 + 0.73 \exp(-0.8 \overline{T}u)] [550 + 680 (1 + \overline{T}u - 21\lambda)^{-1}]$$
 (20)

where

$$\lambda = \frac{\theta^2}{v} \frac{du_s}{dx}$$

$$Tu = \frac{rms \ velocity \ fluctuation \times 100}{u_s}$$

and the mean turbulence level  $\overline{T}u$  characterizes the flow throughout the history of the boundary layer. For  $\lambda=0$ , equation (20) agrees with equation (19) within  $\pm 5$  percent. Very little information exists as to what value of  $\operatorname{Re}_{\theta}$  to use when the change from transition to turbulent flow occurs, but a value of  $\operatorname{Re}_{\theta}=360$  is used (ref. 11).

#### RESULTS AND DISCUSSION

A method which calculates laminar, transitional, and turbulent convective heat-transfer coefficients over the surfaces of turbine vanes is presented. A sample calculation was made for the suction surface of a J-75 size turbine vane whose profile is identical to that given in reference 13. The calculation was made for given gas stream conditions of turbine inlet temperature equal to 1273 K, turbine inlet pressure of 45.7 newtons per square centimeter, fuel/air ratio of 0.0, vane channel inlet Mach number of 0.31, and a vane channel outlet Mach number of 0.84. The vane metal temperature and free-stream dimensionless velocity distributions used in the calculation are given in figure 1. The gas stream conditions given previously were used in the quasi-three-dimensional computer program of reference 14 to calculate the velocities. A short computer program was written to perform the calculation of the heat-transfer coefficients. These velocities were input directly into this program.

The calculated convective heat transfer coefficient distribution for the vane suction surface is shown in figure 2. Also shown in the figure are the distributions obtained from the turbulent flat plate equation and from a finite-difference computer program (ref. 2). The properties of air were used in all cases.

The flat plate distribution is for a flow assumed to be laminar over the cylindrical leading edge and completely turbulent over the remaining vane surface. While closer agreement would be obtained by using the transition criteria with the gas stream velocity constant, the assumption shows the difference between the approximate integral solution and the more conservative (all turbulent) flat plate solution. The approximate integral solution and the finite-difference solution both show distinct regions of laminar, transitional, and turbulent flow.

The value of the momentum thickness Reynolds number for transition from laminar to transitional and from transitional to fully turbulent flow was assumed to be 200 and 360, respectively, for the integral solution. As a second case of the integral solution, the flow was assumed to be fully turbulent near 4.2 centimeters. This position corresponds to a film-cooling slot on the turbine vane profile of reference 13. The assumption of fully turbulent flow was based on the results of reference 15. Experimental temperature data were presented showing a film cooling layer causing a laminar or transitional flow to become turbulent. The finite-difference solution requires no a priori assumption regarding transition. The turbulent viscosity and thermal diffusivity are calculated from the turbulence model (turbulent kinetic energy and mixing length) and added to their respective molecular values. For transitional boundary layers, the turbulent and molecular components are the same order of magnitude.

For purposes of comparison assume that the finite-difference solution gives the baseline heat-transfer coefficient distribution. The turbulent flat plate solution (the conservative solution), except for the vane surface between 4.0 and 6.4 centimeters where

it is less than 10 percent different, is always 50 to 150 percent higher than the baseline. If laminar flow had been assumed at the beginning of the vace, the solution would have been lower than the baseline consistently. The approximate integral solution method, which accounts for the laminar, transitional, and turbulent flow regimes, produced a distribution which is within 15 percent of the baseline between 0.6 and 2.2 centimeters, where both programs predict that laminar flow exists. Between 2.2 and 4.2 centimeters, the integral solution returns transition flow heat-transfer coefficients an average 50 percent higher than paseline. The integral solution which is fully turbulent at the film cooling slot agrees with the baseline within 15 percent from 4.2 to 6.4 centimeters. The other integral solution does not become fully turbulent until 5.2 centimeters. From that point antil 6.4 centimeters, the two integral solutions are in essential agreement. The baseline finite-difference solution goes to fully turbulent flow at 4.2 centimeters. The programmed solution accounted for a film-cooling blowing rate in its calculations. The surface region between 0.5 and 5.2 centimeters includes the two points of change (laminar to transitional and transitional to fully turbulent) and the transitional flow regime itself. The use of the laminar expression (eq. (18)) for the momentum thickness Reynolds number in the approximate integral solutions produces an early change to transitional flow and a later change to turbulent flow than is produced by the finite-difference solution, which uses the calculated boundary layer momentum thickness to arrive at the local momentum thickness Reynolds number. The expression for the transitional regime heat-transfer coefficient (eq. 16), while being of the same level, does not follow the finer details of the heat-transfer coefficient produced by the finite-difference solution near 1.0 centimeter. Figure 2 does imply that the heat-transfer rate to the vane is about equal for the integral and finite-difference solutions.

One possible source of error is the assumption that equation (18) is valid in transitional flow region and up to a value of 360. No assumption at all is made about the accuracy of  $\mathrm{Re}_{\theta}$  as calculated by equation (18) once the value exceeds 360. In addition, no provision for retransition is made.

For the variable surface temperature correction of equations (5a) to (5c), if b is greater than 0.6 or less %an -0.76 it is set equal to the respective limit and  $h_{rat}$  is calculated. For the distribution of figure 2, all values of b were within the specified range. The correction for approximating wedge flow solutions is made according to equation (3). The correction is a function of Eu and  $T_s/T_m$ . If the interpolation for  $F_{\ell am}$  exceeds the range of tabulated values,  $F_{\ell am}$  is set equal to 0.332. Equation (3) then degenerates to the traditional laminar flat plate solution. In figure 2 this degeneration occurred at x = 0.8 centimeter.

The relative value of the approximate integral solution over the turbulent flat plate or the finite-difference solution lies with the user. For the user interested in a conservative order of magnitude value of heat-transfer coefficient, the turbulent flat plate solution and a desk calculator will satisfy his needs. The user of the finite-difference

solution program has the greatest flexibility; he will be able to solve the most difficult problems, such as the prediction of the local surface coefficient of heat transfer near film cooling slots and on the surface of full coverage film cooled vanes. The approximate integral solution method gives the user a heat-transfer-coefficient distribution which takes into approximate account the laminar, transitional, and turbulent flow regions. In situations which do not involve blowing into the boundary layer or severe compressibility effects, the user can obtain heat-transfer-coefficient distributions of acceptable accuracy with relatively little effort and with no need to deal explicitly with turbulent transport parameters. In addition, when adequate computer facilities are not available, the integral technique can be adapted to desk calculators, and it is even feasible for slide rule calculations.

#### SUMMARY OF RESULTS

An approximate integral solution method for calculating laminar, transitional, and turbulent convective heat-transfer coefficients over the surfaces of turbine vanes is described. An example calculation of heat-transfer coefficients using a turbulent flat plate solution, the integral solution, and a finite-difference solution was made for a turbine vane. The results are as follows:

- (1) The approximate integral solution and the finite-difference solution agree well in the laminar and turbulent regions.
- (2) The finite-difference solution predicts a transition region whose length approximates that predicted by the integral solution. Its onset begins much sooner, however.
- (3) Use of either the integral solution or the finite-difference solution in a heat flux calculation would result in substantially lower levels of heat flux to the vame than would the use of the turbulent flat plate solution.

Lewis Research Center,

National Aeronautics and Space Administration, Cleveland, Ohio, October 17, 1977, 198-10.

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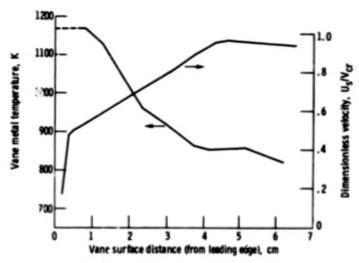


Figure 1. - Vane metal temperature and dimensionless velocity profiles.

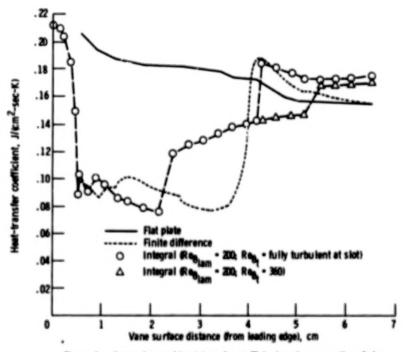


Figure 2. - Comparisons of heat-transfer coefficients using properties of air.

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16	Abstract					
	A method for calculating lamin cients for turbine vane surface uced results in good agreemen	s is described.	An approximate int			
	are presented herein. The int tion results in the laminar and sition produced a later starting regime.	egral solution res turbulent regions	sults agreed well w s. Differences in o	ith the finite-dif	tween the two ference solu- tart of tran-	
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